Michigan

TEST FOR TEACHER CERTIFICATION

# STUDY GUIDE

## 022 Mathematics (Secondary)

MICHIGAN STATE BOARD OF EDUCATION 1849

MI-SG-FLD022-06

## TABLE OF CONTENTS

### **PART 1:** General Information About the MTTC Program and Test Preparation



### **PART 2:** Test Objectives and Sample Test Questions

# PART 1: General Information About the MTTC Program and Test Preparation

The first section of the study guide is available in a separate PDF file. Click the link below to view or print this section.

General Information About the MTTC Program and Test Preparation

# **PART 2:** Test Objectives and Sample Test Questions

## INTRODUCTION

This section includes a list of the test objectives, immediately followed by sample test questions and an answer key for the field covered by this study guide.

### Test Objectives

As noted, the test objectives are broad, conceptual statements that reflect the knowledge, skills, and understanding an entry-level teacher needs in order to teach effectively in a Michigan classroom. Each field's list of test objectives represents the **only** source of information about what a specific test will cover and, therefore, should be studied carefully.

The test objectives are organized into groups known as "subareas." These subareas define the major content areas of the test. You will find a list of subareas at the beginning of the test objective list. The percentages shown in the list of subareas indicate the approximate weighting of the subareas on the test.

### Sample Multiple-Choice Test Questions

The sample multiple-choice test questions included in this section are designed to give the test-taker an introduction to the nature of the test questions included on the MTTC test for each field. The sample test questions represent the various types of test questions you may expect to see on an actual test; however, they are **not** designed to provide diagnostic information to help you identify specific areas of individual strengths and weaknesses or predict your performance on the test as a whole. Use the answer key that follows the sample test questions to check your answers.

To help you identify which test objective is being assessed, the objective statement to which the question corresponds is listed in the answer key. When you are finished with the sample test questions, you may wish to go back and review the entire list of test objectives and descriptive statements once again.

### Mathematics (Secondary) (22) Field-Specific Information

**Approved Graphing Calculators.** Examinees taking the Mathematics (Elementary) or Mathematics (Secondary) test must bring their own graphing calculator but may **not** bring a calculator manual. Graphing calculators will not be provided at the test session. Only the brand and models listed below may be used. **Note that the list of approved graphing calculators for paper-based testing is different than the list for computer-based testing.** Make sure to review the appropriate list for your test. Approved calculator brands and models are subject to change; if there is a change, examinees will be notified. Test administration staff will clear the memory of your calculator both before and after testing. Therefore, be sure to back up the memory on your calculator, including applications, to an external device before arriving at the test center. The list of approved calculators can be found on the Mathematics (Secondary) page at www.mttc.nesinc.com.

# TEST OBJECTIVES

**Calculators for the Mathematics Test**

You may bring your own graphing calculator to the test administration. However, only the brands and models listed on the MTTC website may be used at the administration.

The approved calculator brands and models are subject to change. If there is a change, examinees will be notified.

**If you bring your own calculator, test administration staff will clear the memory of your calculator both before and after the test.** Be sure that you back up the memory on your calculator, including applications, before arriving at the test site.

| Subarea | Approximate Percentage of Questions on Test |
|---|---|
| Mathematical Processes and Number Concepts | 22% |
| Patterns, Algebraic Relationships, and Functions | 28% |
| Measurement and Geometry | 22% |
| Data Analysis, Statistics, Probability, and Discrete Mathematics | 28% |

The appropriate use of technology (e.g., calculators, computers) is integral to the exploration of concepts, skills, and applications in all areas of mathematics. Although technology is mentioned in some test objectives but not in others, the teacher candidate should be aware of the uses and applications of technology across the range of mathematics topics.

## MATHEMATICAL PROCESSES AND NUMBER CONCEPTS

**Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.**

Includes analyzing the nature and purpose of axiomatic systems; using inductive and deductive logic to develop and validate conjectures; applying the laws of deductive logic to draw valid conclusions; developing counterexamples to a conjecture; developing and evaluating direct and indirect proofs; using appropriate mathematical terminology; translating common language into symbols and vice versa; using a variety of numeric, symbolic, and graphic methods to communicate mathematical ideas and concepts; and making connections among numeric, symbolic, graphic, and verbal representations.

**Understand problem-solving strategies, connections among different mathematical ideas, and the use of mathematics in other fields.**

Includes devising, carrying out, and evaluating a problem-solving plan; applying a range of strategies (e.g., drawing a diagram, working backwards, creating a simpler problem) to solve problems; analyzing problems that have multiple solutions; selecting an appropriate tool or technology to solve a given problem; recognizing connections among two or more mathematical concepts (e.g., Fibonacci numbers and the golden rectangle; symmetry and group theory); exploring the relationship between geometry and algebra; and applying mathematics across the curriculum and in everyday contexts.

**Understand number systems and equivalent ways of representing numbers.**

Includes identifying characteristics and relationships among natural, whole, integer, rational, irrational, real, imaginary, and complex numbers (e.g., $\frac{1}{2} = 0.5 = 50\% = \sqrt{\frac{1}{4}}$); applying properties of number operations (e.g., commutative, distributive); applying order relations to numbers; using set operations (e.g., union, intersection, complement); and using manipulatives, verbal expressions, and geometric models to represent numbers.

**Understand number theory and operations on number systems.**

Includes analyzing properties of prime numbers, factors, multiples, and divisibility; extending the relationships of primes, factors, multiples, and divisibility in an algebraic setting; using scientific notation to compute with very large and very small numbers; comparing and contrasting models of operations across number systems (e.g., using a rectangular array to model multiplication of whole numbers and fractions); solving problems involving ratios and proportional reasoning; using manipulatives, verbal expressions, and geometric models to represent number operations; applying and evaluating estimation strategies; analyzing standard and nonstandard computational algorithms; solving a variety of problems using number operations; performing operations with complex numbers (e.g., conjugates, products, roots); and using rectangular, polar, matrix, and vector representations to solve problems.

## PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS

**Describe, analyze, and generalize mathematical patterns.**

Includes recognizing and extending numerical and geometric patterns; constructing, representing, and recording patterns using charts, tables, graphs, matrices, and vectors; exploring and describing symmetric and spatial patterns (e.g., fractals, tessellations); analyzing and generalizing sequences, series, and recursive patterns; and using patterns to make inferences, predictions, and decisions.

**Use symbolic expressions to describe and analyze patterns of change and functional relationships.**

Includes exploring patterns of change characteristic of families of functions (e.g., quadratic, exponential, periodic); translating among verbal, graphic, tabular, and symbolic representations of functions; distinguishing between relations and functions; analyzing functions in terms of range, domain, and intercepts; exploring function operations [e.g., $f(x) + g(x)$], composition [e.g., $f(g(x))$], and inverses; using piecewise- and recursively defined functions; analyzing the relationship among the graphs of $f(x)$ and transformations such as $f(x \pm c)$, $cf(x)$, and $\frac{1}{f(x)}$; and using graphing calculators and utilities to analyze properties of functions.

**Understand properties and applications of linear and quadratic functions, and solve related equations and inequalities.**

Includes analyzing linear relationships; modeling and solving problems using linear equations and inequalities; investigating the relationship between a linear equation and its graph; modeling and solving problems using linear systems (e.g., using matrices, using graphs); solving quadratic equations, inequalities, and systems using a variety of methods (e.g., graphical, analytic); using graphing calculators to solve systems of equations; analyzing how changing the coefficients of a quadratic function changes its graph; and using quadratic functions to model and solve problems, including maximum and minimum problems.

**Understand properties and applications of polynomial, rational, radical, exponential, logarithmic, and trigonometric functions, and solve related equations and inequalities.**

Includes exploring the properties and graphs of polynomial, rational, radical, exponential, logarithmic, and trigonometric (i.e., sine, cosine, tangent) functions; applying these functions to develop and evaluate models of real-world situations; modeling and solving problems using polynomial, rational, radical, exponential, logarithmic, and trigonometric equations and inequalities; analyzing the relationship between exponential and logarithmic functions; examining the relationship between trigonometric functions and their inverses; examining the relationship between trigonometric functions and circular functions; and modeling periodic phenomena using trigonometric functions.

**Understand principles and applications of differential and integral calculus.**

Includes investigating limits and limiting processes; using limits to determine continuity; analyzing the relationships among the graph, slope of the secant line, and the derivative of a function; using differential calculus to analyze the graph of a function; analyzing the relationship among the area under a curve, Riemann sums, and integration; and using the principles of calculus and appropriate technology to solve a variety of theoretical and applied problems.

## MEASUREMENT AND GEOMETRY

**Understand attributes of measurement and measuring units.**

Includes selecting appropriate units to estimate and record measurements of angle (degree and radian), length, area, volume, mass, temperature, and time; identifying tools for performing measurements; converting measurements within measurement systems; analyzing how changes in the measurement of one attribute relate to changes in others; using dimensional analysis to solve problems; solving problems involving density, pressure, rates of change, and other derived units; and evaluating precision, accuracy, measurement errors, and percent error.

**Apply measurement principles to analyze the spatial characteristics of two- and three-dimensional shapes.**

Includes deriving and applying formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures; exploring scale factors for the area and volume of similar figures; applying right triangle trigonometry and the Pythagorean theorem to solve problems (e.g., problems involving indirect measurements); interpreting three-dimensional drawings of objects; and analyzing cross sections and nets of three-dimensional figures.

**Apply geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of two- and three-dimensional shapes.**

Includes determining necessary and sufficient conditions for the existence of a particular shape; analyzing concepts (e.g., parallelism) in Euclidean and non-Euclidean geometries; applying properties of parallel and perpendicular lines and angles to analyze shapes; comparing and analyzing shapes and formally establishing the relationships among them (e.g., congruence, similarity); using geometric principles to prove theorems; applying properties of two-dimensional shapes to analyze three-dimensional shapes; and recognizing the uses of dynamic geometry software in making conjectures and investigating properties of shapes.

**Apply properties of geometric transformations and coordinate and vector methods to describe geometric objects in two and three dimensions.**

Includes analyzing figures in terms of translations, reflections, rotations, dilations, and contractions; applying transformations to explore the concepts of congruence and similarity; using transformations to characterize the symmetry of an object; representing transformations using matrices; analyzing the composition and inverse of transformations; describing the abstract algebraic properties of a set of transformations under composition; locating objects in terms of their position using rectangular, polar, and three-dimensional coordinate systems; locating and describing the locus of points that satisfy a given condition; applying concepts of slope, distance, midpoint, and parallel and perpendicular lines to determine the geometric and algebraic properties of figures in the coordinate plane (including conic sections); and describing the position and movement of objects using vectors.

## DATA ANALYSIS, STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS

**Understand methods of collecting, organizing, and displaying data.**

Includes formulating questions requiring data gathering and applying appropriate techniques for collecting data; analyzing factors that may affect the validity of a survey, including bias; organizing data using tables and spreadsheets; creating a variety of charts to display data (e.g., pie charts, box plots, stem-and-leaf plots, scatter plots, frequency histograms); using appropriate technology to organize and display data; and evaluating the source, organization, and presentation of data.

**Understand methods of describing, analyzing, and interpreting data.**

Includes analyzing the shape, location, and spread of a data distribution using algebraic and geometric methods to estimate a variety of statistics; describing the range and outlines of a set of data; applying and interpreting measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation); analyzing the effects of data transformations on measures of central tendency and spread; finding the function (e.g., linear, exponential, logarithmic) that best represents a set of data; using appropriate technology to analyze and manipulate data; and evaluating the validity of statistical arguments.

**Understand methods of making predictions and inferences based on data.**

Includes analyzing and explaining data trends; making and testing hypotheses; using simulations and sampling to test inferences; applying principles of interpolation and extrapolation; analyzing linear regression lines and correlation coefficients; analyzing the relationship between sample size and width of confidence interval; and employing confidence intervals in making predictions and inferences based on data.

**Understand the theory of probability and probability distributions.**

Includes enumerating the sample space of an event; determining simple and compound probabilities; finding the probability of dependent and independent events; using simulations and sampling to determine experimental probabilities; solving problems using geometric probability (e.g., ratio of two areas); applying probability distributions (e.g., binomial, normal) to solve problems; and modeling and solving real-world problems using probability concepts.

**Understand principles of discrete mathematics.**

Includes solving counting problems using permutations and combinations; using sets and set relations to represent algebraic and geometric concepts; using vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees, and adjacency matrices; proving statements using the principle of mathematical induction; employing recursion and iteration methods to model problems; describing and analyzing efficient algorithms to accomplish a task or solve a problem in a variety of contexts (e.g., practical and computer-related situations); using discrete mathematics concepts to model a problem, evaluate the existence of solutions, determine the number of possible solutions, and choose the optimal solution to the problem; and using linear programming to model and solve problems.

## SAMPLE MATHEMATICS FORMULAS

| Formula | Description |
|---|---|
| $V = \frac{1}{3}Bh$ | Volume of a right circular cone and a pyramid |
| $A = 4\pi r^2$ | Surface area of a sphere |
| $V = \frac{4}{3}\pi r^3$ | Volume of a sphere |
| $A = \pi r\sqrt{r^2 + h^2}$ | Lateral surface area of a right circular cone |
| $S_n = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$ | Sum of an arithmetic series |
| $S_n = \frac{a(1 - r^n)}{1 - r}$ | Sum of a geometric series |
| $\sum_{n=0}^{\infty} ar^n = \frac{a}{1 - r},\ \lvert r \rvert < 1$ | Sum of an infinite geometric series |
| $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | Distance formula |
| $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ | Midpoint formula |
| $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ | Slope |
| $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ | Law of sines |
| $c^2 = a^2 + b^2 - 2ab \cos C$ | Law of cosines |
| $s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n - 1}$ | Variance |
| $s = r\theta$ | Arc length |
| $x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Quadratic formula |

## SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

**Calculators for the Mathematics Test**

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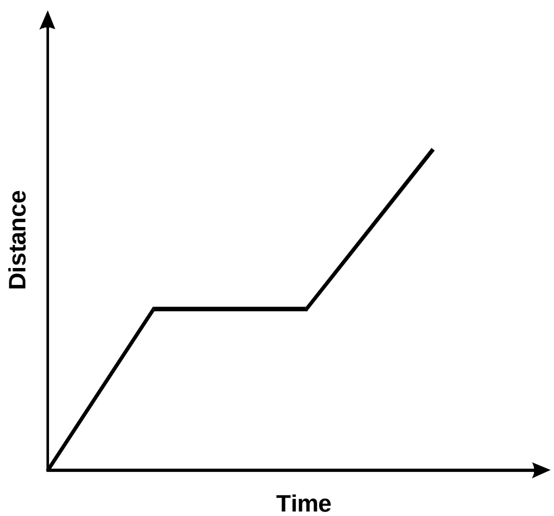
1. Use the graph below to answer the question that follows.




Students in a class are making qualitative graphs of their motion as they move along a straight line. Which of the following statements best describes the motion represented in the distance versus time graph above?

A. The student accelerated to a certain speed, walked at that speed for a period of time, and then started accelerating again.

B. The student started walking at a constant speed, then stopped for a while before continuing at approximately the same speed.

C. The student walked along a straight line, turned right, continued walking in a straight line, turned left, and then continued walking in a straight line.

D. The student started speeding up, stopped for a period of time, and then resumed walking at a faster and faster speed.

2. While studying transformations of geometric figures, the students in a geometry class made the following observations about rotations of geometric figures.

> The composition of two rotations with the same center is a rotation.
>
> If $R_1$, $R_2$, and $R_3$ are rotations with the same center, then $(R_1 * R_2) * R_3 = R_1 * (R_2 * R_3)$
>
> Rotation by an angle $\theta = 0$ leaves any object unchanged.
>
> For every rotation there exists an inverse rotation that undoes the effect of the initial rotation.

A teacher could use these observations to show the connections between geometry and algebra by discussing which of the following algebraic structures?

A. groups

B. fields

C. rings

D. vector spaces

3. Use the sets below to answer the question that follows.

Universal set is $U = \{x \mid x \text{ is a real number}\}$ and
$S = \{x \mid x \text{ is of the form } \frac{a}{b}\ (b \neq 0) \text{ where } a \text{ and } b \text{ are integers}\}$

Which set is the complement of *S*?

A. $\{x \mid x \text{ is a prime number}\}$

B. $\{x \mid x \text{ is an irrational number}\}$

C. $\{x \mid x \text{ is a real number}\}$

D. $\{x \mid x \text{ is an integer}\}$

4. Use the diagram below to answer the question that follows.

| | |
|---|---|
| $x^2$ | $8x$ |
| $2x$ | 16 |

The diagram above is a geometric representation of which of the following operations?

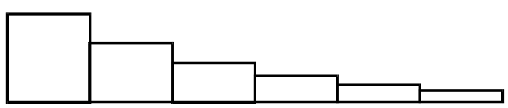
A. the sum of $(x + 4)^2$ and $10x$

B. the difference of $(x – 4)^2$ and $2x$

C. the product of $(x + 2)$ and $(x + 8)$

D. the quotient of $x^2 + 16$ and $10x$

5. Use the diagram below to answer the question that follows.



In the sequence of rectangles shown above, the first rectangle is a square of area one. Each successive rectangle has the same base, but has two-thirds the height of the previous rectangle. If this pattern is continued indefinitely, which series represents the sum of the areas of all the rectangles?

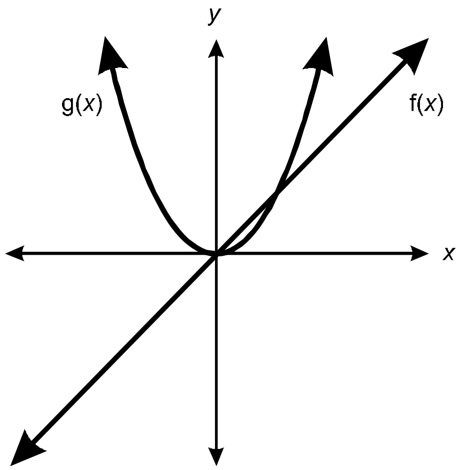
A. $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \ldots$

B. $1 + \frac{2}{3} + \frac{4}{6} + \frac{6}{9} + \frac{8}{12} + \ldots$

C. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$

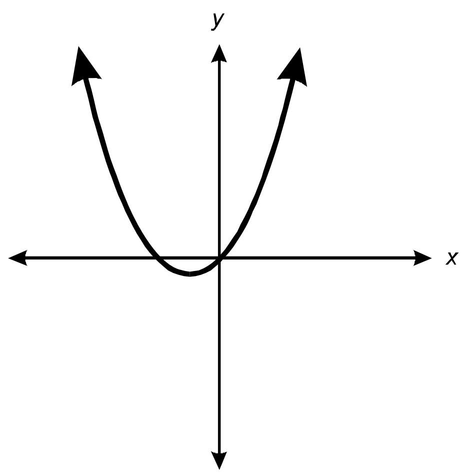
D. $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \ldots$

6. Use the graphs of the functions below to answer the question that follows.

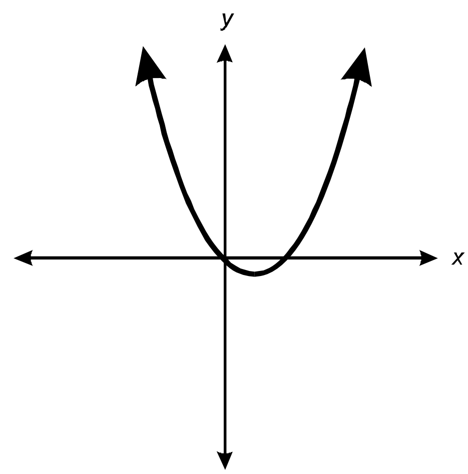


Given the two functions above, which of the following graphs represents the sum of the two functions?

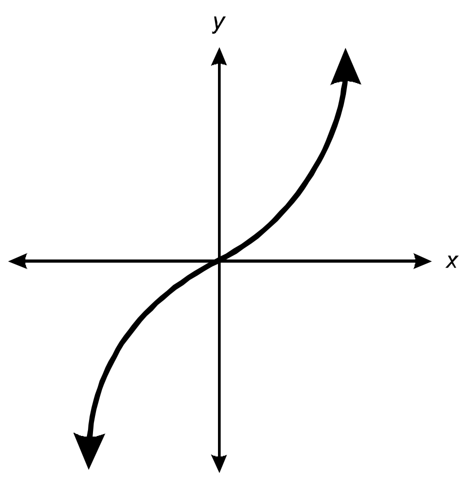
A.



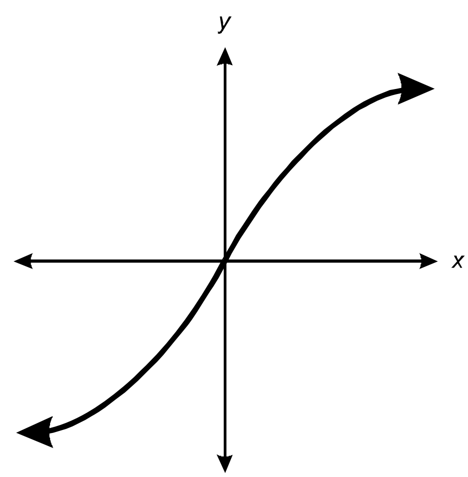

B.




C.




D.



7. Use the table below to answer the question that follows.

| Location | Altitude (feet) | Boiling Point of Water (°F) |
|---|---|---|
| Sea Level | 0 | 212 |
| Denver, CO | 5,000 | 203 |

The temperature at which water boils is a function of altitude above sea level. How well does a linear model based on the data in the table predict the temperature (160°F) at which water boils on Mt. Everest (height = 29,000 feet)?

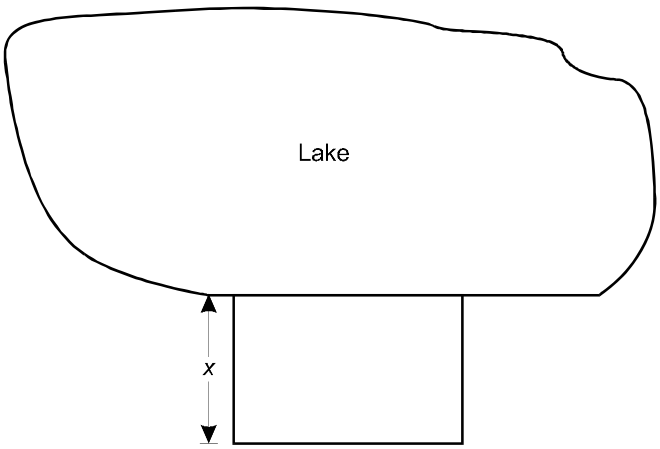
A. very well, with an error less than 1%

B. sufficiently well, with an error of about 4%

C. good enough for a rough estimate, with an error of about 8%

D. very poorly; a linear model should not be used to characterize this relationship

8. Use the diagram below to answer the question that follows.




A person is fencing in three sides of a rectangular region using the straight part of a lake shoreline as the fourth side of the region. No fencing is required for the fourth side. The person has 110 meters of fencing, and the area of the region is 1,200 square meters. Which of the following quadratic equations should be solved to find the value of $x$?

A. $x^2 + 110x + 1200 = 0$

B. $x^2 - 55x + 600 = 0$

C. $x^2 - 55x + 1200 = 0$

D. $x^2 - 110x + 600 = 0$

9. A wildlife biologist discovers that a bird population in a certain area is being reduced by half every 6.3 years. The population will be unable to sustain itself if it drops below its lower limit of viability, which is 1,500. If the original population of birds is 5,000, how long will it take for the population to reach its lower limit of viability?

A. 6.3 years

B. 8.8 years

C. 10.9 years

D. 11.3 years

10. An engineer would like to find the lowest degree polynomial that passes through the points (⁻4, 0), (⁻2, 0), (1, 0), and (3, 0). Which of the following functions satisfies the engineer's requirement?

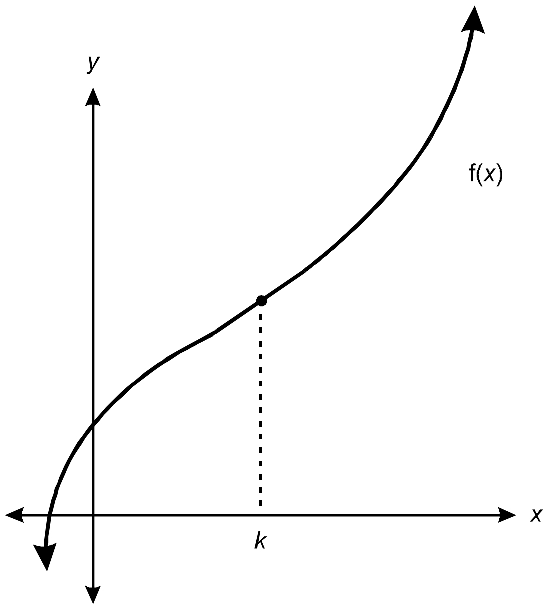
A. $f(x) = (x - 4)(x - 2)(x + 1)(x + 3)$

B. $f(x) = (x + 4)(x + 2) + (x - 1)(x - 3)$

C. $f(x) = (x + 4)(x + 2)(x - 1) + (x - 3)$

D. $f(x) = (x + 4)(x + 2)(x - 1)(x - 3)$

11. Use the graph below to answer the question that follows.




If $f(x) = ax^3 + bx^2 + cx + d$, which of the following equations should be solved to find the value of $k$?

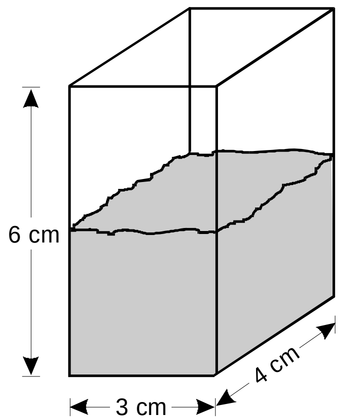
A. $6ax + 2b = 0$

B. $3ax^2 + 2bx + c = 0$

C. $ax^3 + bx^2 + cx + d = 0$

D. $\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx + e = 0$

12. Use the diagram below to answer the question that follows.




When an irregularly shaped object of mass 22.0 g is dropped into the rectangular container shown above, the object sinks to the bottom and the water level rises 1.5 cm. What is the density of the object, rounded to the nearest tenth?

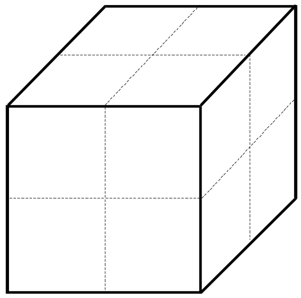
A. 0.8 g/cm$^3$

B. 1.2 g/cm$^3$

C. 1.8 g/cm$^3$

D. 14.7 g/cm$^3$

13. Use the diagram below to answer the question that follows.



A cube has a surface area of 216 square inches. The dotted lines shown bisect the sides of the cube. Each side of the cube is sliced along the dotted lines to produce a number of smaller cubes. What is the sum of the surface areas of all the cubes produced?

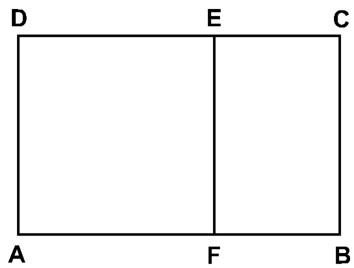
A. 216 in.$^2$

B. 432 in.$^2$

C. 864 in.$^2$

D. 2,592 in.$^2$

14. Use the diagram below to answer the question that follows.




Rectangles *ABCD* and *BCEF* are similar, *AF* = *AD*, and $\overline{EF}$ is perpendicular to $\overline{AB}$. If *AB* = 1 and *FB* = *x*, which of the following equations must be true?

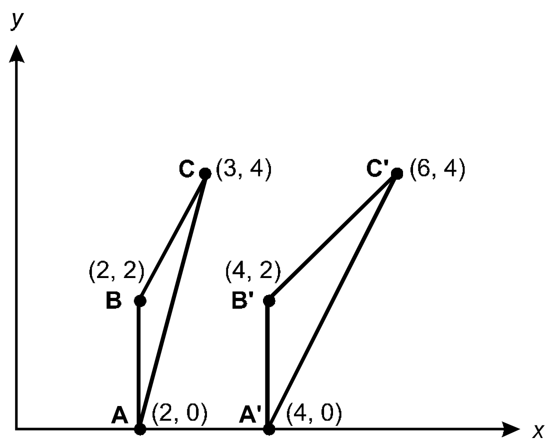
A. $x^2 - x + 1 = 0$

B. $x^2 - 2x - 2 = 0$

C. $x^2 - 2x + 2 = 0$

D. $x^2 - 3x + 1 = 0$

15. Use the diagram below to answer the question that follows.




The transformation $T$ maps each point on $\triangle ABC$ to a point on $\triangle A'B'C'$. Which matrix represents $T$?

A. $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

16. A group of high school students is planning a survey to determine the type of music students in their school listen to. To obtain the most statistically valid results, which of the following data collection techniques should the students employ?

    A. Questionnaires should be made available at all study halls for students to fill out and hand in at their own convenience.

    B. A sample of students should be chosen at random from class lists and interviewed individually.

    C. Students seen listening to music with headphones or other devices should be interviewed about what type of music they like.

    D. The students in the group should ask their friends about the types of music they prefer.

17. When a set of data $(x, y)$ is plotted on a standard coordinate system, the data points essentially lie on the arc of a nonlinear curve. When the data are transformed by taking the natural logarithm of the $y$-values and graphed as $(x, \ell n\ y)$, the transformed data are close to lying on a straight line. This indicates the initial data would be best modeled by:

    A. an exponential function.

    B. a quadratic function.

    C. a rational function.

    D. a square root function.

18. An instructional computer simulation starts with a population consisting of {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. The mean of this population is 4.5 with a standard deviation of 2.87. The computer determines the mean of each of the 252 samples of size 5 and displays the distribution of the sample means as a histogram. The computer then compares this histogram with a normal curve having mean of 4.5 and standard deviation of $\frac{2.87}{\sqrt{5}}$. What is the primary purpose of this demonstration?

    A. to demonstrate the central limit theorem

    B. to compare and contrast the mean and the median of a distribution

    C. to demonstrate how to find standard *z* scores for the normal curve

    D. to demonstrate the effect of data transformations on the mean and standard deviation

19. A target consists of a circle inscribed in a square. If a dart is randomly thrown at the square, what is the probability to the nearest thousandth that it will land inside the inscribed circle (assume the dart lands inside the square)?

    A. 0.318

    B. 0.637

    C. 0.785

    D. 0.858

20. A volunteer organization has set up a phone tree to reach all of its members quickly. In level one of the tree, the first person calls three members of the organization. In level two, each of the three people call three other members. If this patterns continues, how many people would have been called after *n* levels?

A. $3^n$

B. $n^3$

C. $\sum_{k=1}^{n} k^3$

D. $\sum_{k=1}^{n} 3^k$

## ANSWER KEY FOR THE SAMPLE MULTIPLE-CHOICE TEST QUESTIONS

| Item Number | Correct Response | Objective |
|---|---|---|
| 1. | **B** | Understand principles of mathematical reasoning and techniques for communicating mathematical ideas. |
| 2. | **A** | Understand problem-solving strategies, connections among different mathematical ideas, and the use of mathematics in other fields. |
| 3. | **B** | Understand number systems and equivalent ways of representing numbers. |
| 4. | **C** | Understand number theory and operations on number systems. |
| 5. | **D** | Describe, analyze, and generalize mathematical patterns. |
| 6. | **A** | Use symbolic expressions to describe and analyze patterns of change and functional relationships. |
| 7. | **A** | Understand properties and applications of linear and quadratic functions, and solve related equations and inequalities. |
| 8. | **B** | Understand properties and applications of linear and quadratic functions, and solve related equations and inequalities. |
| 9. | **C** | Understand properties and applications of polynomial, rational, radical, exponential, logarithmic, and trigonometric functions, and solve related equations and inequalities. |
| 10. | **D** | Understand properties and applications of polynomial, rational, radical, exponential, logarithmic, and trigonometric functions, and solve related equations and inequalities. |
| 11. | **A** | Understand principles and applications of differential and integral calculus. |
| 12. | **B** | Understand attributes of measurement and measuring units. |
| 13. | **B** | Apply measurement principles to analyze the spatial characteristics of two- and three-dimensional shapes. |
| 14. | **D** | Apply geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of two- and three-dimensional shapes. |
| 15. | **C** | Apply properties of geometric transformations and coordinate and vector methods to describe geometric objects in two and three dimensions. |
| 16. | **B** | Understand methods of collecting, organizing, and displaying data. |
| 17. | **A** | Understand methods of describing, analyzing, and interpreting data. |
| 18. | **A** | Understand methods of making predictions and inferences based on data. |
| 19. | **C** | Understand the theory of probability and probability distributions. |
| 20. | **D** | Understand principles of discrete mathematics. |